ON A MODEL OF FLUID MOTION NEAR A SOLID SURFACE

V. S. Shorkin

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The possibility is examined of using the model of micropolar media [1] to describe the anomalous change in the mechanical properties and the associated appearance of the scale effect in the boundary layers of certain fluids.

1. It is shown in a number of papers [2-6] that the surface of a solid is capable of altering the structurally sensitive properties of adjacent fluid layers on the order of 10^{-5}-10^{-6} cm thick. The action of the solid surface on the structure of the fluid is transmitted from layer to layer because of the approximate influence of epitaxial centers and of different intermolecular forces, particularly the most powerful, the hydrogen bond. Consequently, the heat conduction, shear elasticity, viscosity, and a number of other properties vary, and the molecules acquire a definite spatial orientation. The data presented are considered [3] as an indicator of a special boundary phase configuration for which the mechanism of formation is similar to the corresponding mechanism for the formation of liquid crystals [7]. The presence of a special boundary phase configuration is noted in polar fluids, in whose structure the hydrogen bond plays a significant part, and also in nonpolar fluids containing polar molecule impurities [6].

According to [8], besides the translation velocity vector **u**, it is necessary to use the microrotation velocity vector $\boldsymbol{\omega}$ that is kinematically independent of **u** for the characteristics of a moving fluid in the general case. Let V be a volume element of a liquid medium, N the quantity of molecules it contains, and $\mathbf{L}^i$ (i = 1, 2,..., N) the unit vector characterizing the direction of the major axis of the i-th molecule. Then the angular velocity of an individual molecule can be expressed in terms of $\mathbf{L}^i$ by using the equality [7]

$$\boldsymbol{\omega}^i = \mathbf{L}^i \times \frac{d\mathbf{L}^i}{dt} + a^i\mathbf{L}^i,$$

where a^i is the projection of the vector $\boldsymbol{\omega}^i$ in the direction of the vector $\mathbf{L}^i$, and t is the time. The vector of the mean microrotation velocity $\boldsymbol{\omega}$ can afterwards be determined in the volume V by using the expression

$$\boldsymbol{\omega} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\omega}^i = \frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{L}^i \times \frac{d\mathbf{L}^i}{dt} + a^i\mathbf{L}^i\right).$$

When all the vectors $\boldsymbol{\omega}^i$ agree in direction, i.e., $\mathbf{L}^i = \mathbf{L}$, we can write

$$\boldsymbol{\omega} = \mathbf{L} \times \frac{d\mathbf{L}}{dt} + a\mathbf{L}. \tag{1.1}$$

Here $a = \frac{1}{N}\sum_{i=1}^{N} a^i$ is the mean value of the angular velocity projections of the individual molecules in the direction of their preferred orientation characterized by the vector **L**. The expression (1.1) agrees with the definition of the velocity of microrotation in the hydrodynamics of liquid crystals [7]. It is impossible to extract the preferred molecule orientation in an isotropic Newtonian medium, and the vector $\boldsymbol{\omega}$ agrees with half the curl of the translational velocity

$$\boldsymbol{\omega} = \frac{1}{2}\operatorname{rot}\ \mathbf{u}. \tag{1.2}$$

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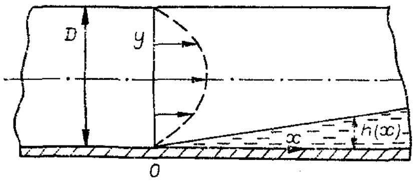

Fig. 1

Taking the above into account, it can be assumed that the properties of a surfactant fluid change with distance from the phase interface from properties similar to the properties of a liquid crystal to properties characteristic for Newtonian media. The vector **ω** here changes from the value determined by (1.1) to a value determined by (1.2). It must also be taken into account that the boundary structure of the fluid can be ruptured partially under the effect of a pressure gradient [3], therefore, (1.1) may not be satisfied even on the phase interface.

The reasoning presented is the basis for the hypothesis that fluid motion near the phase interface can be described in certain cases on the basis of the model of micropolar media [1], whose basic feature is the utilization of the concept of a couple stress, kinematic independence of the vectors **ω** and **u**, and also the absence of the vector **L** among the dynamical variables. To confirm the hypothesis expressed, an attempt is made to describe the anomalous behavior of the boundary layers of certain fluids when studying their mechanical properties by the blowing method.

2. The crux of the effect is the following [9-12]. A plate, on which a wedgelike layer of the fluid being investigated is superimposed (Fig. 1) and whose thickness h is much less than the width of the channel D, is inserted in a flat channel of width D. Then, the fluid is blown for a time by a gas stream moving under the effect of a constant pressure gradient. We will consider [9] that the fluid motion is plane-parallel, and the magnitude of the tangential stress on its open surface equals the value σ of the tangential stress developed by the gas stream on the channel wall in the absence of the fluid. Let us place the origin (the point 0 in Fig. 1) or a rectangular coordinate system on the leading edge of the wedgelike layer of fluid, and let us direct the 0x axis along, and the 0y axis perpendicular to, the streamlined surface. At the initial instant let the equation of the liquid film profile have the form

$$x = x(h).$$

According to the hypothesis on the nature of the flow, after a sufficiently long time segment τ the profile should take the form described by the dependence

$$x = x(h) + u(h)\tau = \tau[u(h) + x(h)/\tau] \simeq \tau u(h),$$

where u(h) is the velocity of the translational motion of the fluid being blown along the solid surface at y = h. For a Newtonian fluid it is possible to obtain

$$x = \tau Dh/\eta,$$

where η is the coefficient of volume viscosity of the fluid being studied. Therefore, the final shape of the blown film profile should be rectilinear. However, the results of experiments with surface-inert Vaseline oil [10], surfactant polydimethylsiloxane oligomer [11], and 2% solution of rubber in Vaseline [12] correspond to the classical representations only in the first case. The profile of the rubber solution on the metal plate turns out to be convex, while the oligomer profile is convex on glass but concave on metal. The distinction from classical properties is most significant near the plate, and the profiles become rectilinear with distance away. If it is assumed [10] that

$$dh/dx = \eta_*/\tau D,$$

where η_* is the coefficient of apparent viscosity, then for τ = const, D = const the slope of the tangent to the profile is proportional to the load value of the coefficient of apparent viscosity. Taking this into account, the profile curvature, noted in [11, 12], indicates the dependence of the apparent viscosity on the distance to the wall.

3. For a stationary plane-parallel flow, the equations of motion of an incompressible micropolar medium can be written in the form [1]

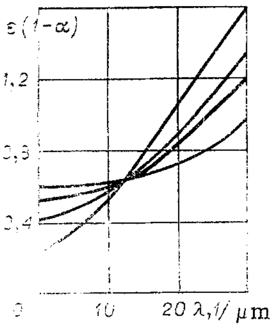

Fig. 2

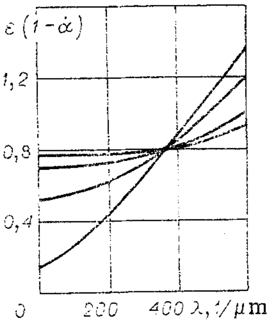

Fig. 3

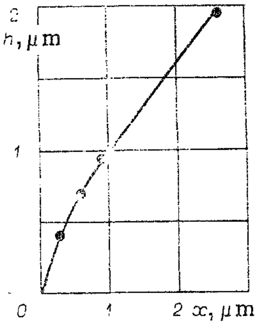

Fig. 4

$$\left(\eta+\frac{\gamma}{4}\right)\frac{d^2u}{dy^2}+\frac{\gamma}{2}\frac{d\omega}{dy}=0,\quad \theta\frac{d^2\omega}{dy^2}-\gamma\left(\omega+\frac{1}{2}\frac{du}{dy}\right)=0, \tag{3.1}$$

where ω is the microrotation velocity around an axis perpendicular to the plane of motion, and γ, θ are viscosity constants $\gamma > 0$, $\theta > 0$. Since none of the experiments listed above displayed slipping of the fluid, we shall assume

$$u=0 \qquad \text{for} \quad y=0. \tag{3.2}$$

The second boundary condition on the solid surface is assumed written in the form

$$\sigma=\eta_* du/dy, \quad \eta_*=\beta\eta \qquad \text{for} \quad y=0. \tag{3.3}$$

Taking the above into account, the conditions

$$\beta=1, \quad \beta>1, \quad \beta<1 \qquad \text{for} \quad \eta_*=\eta, \quad \eta_*>\eta, \quad \eta_*<\eta \tag{3.4}$$

should be satisfied in the equality written for the rectilinear, convex, and concave profiles, respectively. Using the constitutive relationships for the tangential stresses, which have the following form in the case considered [1]

$$\sigma=\left(\eta+\frac{\gamma}{4}\right)\frac{du}{dy}+\frac{\gamma}{2}\omega,$$

the boundary condition (3.3) can be rewritten as follows:

$$\omega=-\alpha\sigma/2\eta,\ \alpha=1+(1-\beta)/\varepsilon\beta,\ \varepsilon=\gamma/(4\eta+\gamma) \quad \text{for} \quad y=0. \tag{3.5}$$

Here the constant α determines the value of the microrotation velocity in fractions of the classical angular velocity. Since the fluid motion occurs under the effect of a gas flow which is considered a Newtonian medium, the second pair of boundary conditions is written in the form

$$\left(\eta+\frac{\gamma}{4}\right)\frac{du}{dy}+\frac{\gamma}{2}\omega=\sigma,\quad \frac{d\omega}{dy}=0 \qquad \text{for} \quad y=h. \tag{3.6}$$

Using the solution of the problem (3.1), (3.5), (3.6), the equation of the profile of the liquid film being blown upon can be obtained in the form

$$x=\frac{1}{\Phi}\left[h-\frac{\varepsilon}{\lambda}(1-\alpha)\,th\,(\lambda h)\right],\quad \Phi=\eta/\tau D,\quad \lambda^2=(1-\varepsilon)\gamma/\theta. \tag{3.7}$$

It is seen from (3.7) that if $\alpha < 1$, then the profile is convex, and $\eta_* = \tau D dh/dx > \eta$; if $\alpha = 1$, then the profile is rectilinear and agrees with a Newtonian profile, and $\eta_* = \eta$; if $\alpha > 1$, then the profile is concave and $\eta_* < \eta$. At the same time, according to (3.5) those values of α correspond to the listed values of β which are in conformity with the relationships between η_* and η on the basis of (3.4) written with the elucidated experimental data taken into account, and which are qualitative confirmation of the correctness of the selection of the model of micropolar media for the description of fluid motion near the phase interface.

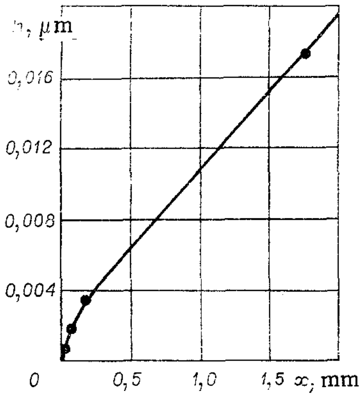

Fig. 5

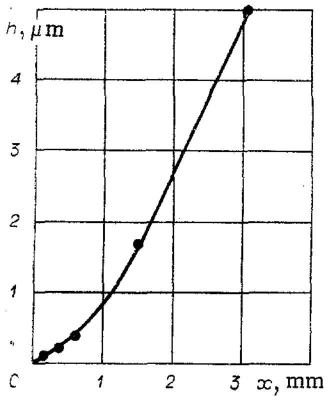

Fig. 6

4. The equations

$$F_k(\varepsilon(1-\alpha); \lambda) = 0, \quad k = 1, 2, 3, 4 \tag{4.1}$$

are constructed for four points on the graph of the dependence $h = h(x)$ obtained experimentally for an oligomer on a glass plate and for a rubber solution in Vaseline oil. Since $\varepsilon(1-\alpha)$ and λ characterize the fluid properties, a single pair of values of these quantities should exist that satisfies the system (4.1), which can be indicative of the independence of the viscosity constant of the micropolar media model from the scale of the experiment being performed. As a result of the solutions of (4.1) (Fig. 2 is the solution for an oligomer, and Fig. 3 for the rubber solution), it is obtained that the quantities $\varepsilon(1-\alpha)$ and λ are independent of the selection of the experimental points, where $\varepsilon(1-\alpha) = 0.65$, $\lambda = 14.2$ 1/μ for glass, and $\varepsilon(1-\alpha) = 0.81$, $\lambda = 400$ 1/μ for the rubber solution on metal. Theoretical profiles of the films blown upon are constructed (the continuous line in Fig. 4 for the oligomer, and in Fig. 5 for the rubber solution) for the values found for $\varepsilon(1-\alpha)$ and λ, which agree satisfactorily with the experimental points.

Since only the conditions on the boundary vary in blowing a film of polydimethylsiloxane oligomer from a metal plate as compared with blowing it from a glass plate, then on a metal plate the parameter $\varepsilon(1-\alpha)$ should satisfy the condition

$$\varepsilon(1-\alpha) = \text{const}, \quad \alpha > 1 \tag{4.2}$$

for the value of λ obtained on glass where $\alpha < 1$. Condition (4.2) is verified for five points of the experimental profile, as noted in Fig. 6, which are taken in order of the distance from the origin. Consequently, it turns out that for these points $\varepsilon(1-\alpha) = -2.32; -2.42; -2.18; -2.36; -2.36$, i.e., condition (4.2) is satisfied.

Therefore, the results elucidated above for processing experimental data on a study of the viscosity of a polydimethylsiloxane oligomer, a rubber solution in Vaseline oil, and Vaseline oil near a solid surface by the method of blowing on the basis of the micropolar media model confirm the hypothesis about the possibility of applying this model to take account of the influence of the phase interface on the mechanical properties of adjoining fluid layers.

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